

# Vectors

## What's a vector?

A **vector** is a quantity that has both a **magnitude** and a **direction**.

An example of a vector quantity is a force. Forces must have both a magnitude (e.g. 17 N) and a direction (e.g. north-east).

## What isn't a vector?

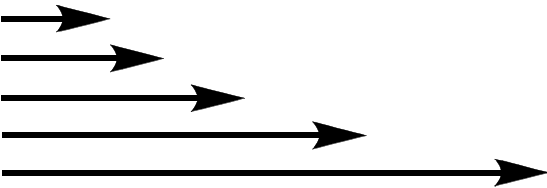
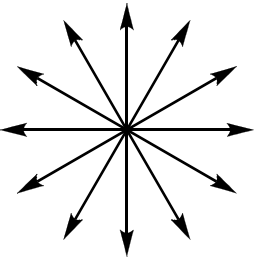
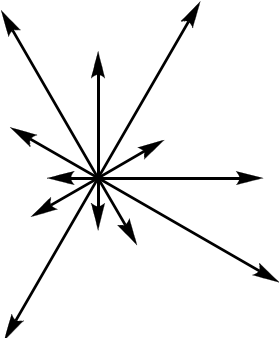
A quantity that has **only magnitude** but no direction is called a **scalar**.

An example of a scalar quantity is a temperature. Temperatures only have a magnitude (e.g. 23 °C). You'll never see a vector temperature – 120 °C south would be meaningless!

## Representing vectors with arrows

We normally represent vectors with arrows – the arrowhead shows the direction of the vector and the length of the arrow shows its magnitude.

If two vectors have the same magnitude and direction, even if they start in different positions, they are equal.

	These vectors all have the same direction but different magnitudes.
	These vectors all have the same magnitude but different directions.
	These vectors all have different magnitudes and different directions.

## Vectors in print and handwriting

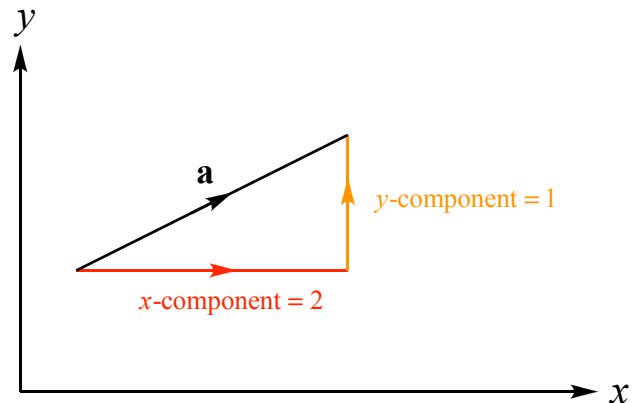
Vectors can be represented in print as bold symbols, such as **a**, or as two letters with an arrow above them, as in  $\vec{OA}$ . The first letter represents the start point of the vector and the second letter represents its end point.

When writing vectors by hand, don't spend ages making your symbols bold! Instead, draw a straight or wavy line underneath the symbol. So you would write a or a instead of **a**.

## Components of a vector

We often describe vectors in terms of their x-components and y-components.

Assuming you have a sheet of paper with x- and y-axes labelled, any vector you draw will have an x-component and a y-component. The x-component is how far the vector moves to the right and the y-component is how far the vector moves upwards.



Finding out the components of a vector is called **resolving a vector into its components**.

## Column vectors

A column vector is a way of representing a vector by displaying its components in a column surrounded by square brackets.

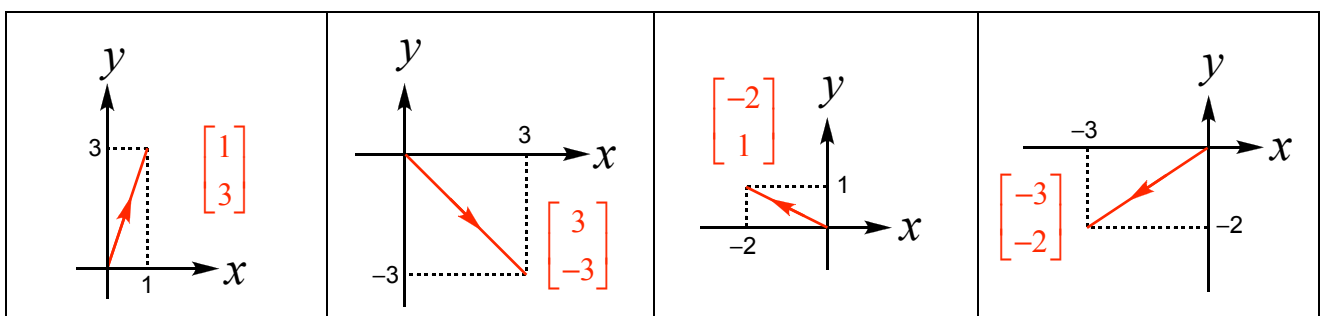
$$\begin{bmatrix} x\text{-component} \\ y\text{-component} \end{bmatrix}$$

In the example above, the x-component is 2 and its y-component is 1.

We can therefore write  $\mathbf{a} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ .

The components of a vector are sometimes denoted by the same letter as the vector itself, but not bold and with a subscript x or y to indicate whether it is the x-component or the y-component.

$$\begin{bmatrix} a_x \\ a_y \end{bmatrix}$$



## Modulus of a vector

The length of a vector is called its modulus. If  $\mathbf{a}$  is a vector, its modulus is denoted by  $|\mathbf{a}|$  or simply  $a$  (not bold).

If you know the components of a vector, you can calculate its length as follows:

If a vector's  $x$ -component is  $a_x$  and its  $y$ -component is  $a_y$  then its modulus is given by  $|\mathbf{a}| = \sqrt{a_x^2 + a_y^2}$ .

Using the example of vector  $\mathbf{a}$  from above,

$$\mathbf{a} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \text{therefore} \quad |\mathbf{a}| = \sqrt{2^2 + 1^2} = \sqrt{4 + 1} = \sqrt{5} \approx 2.24$$

This method is really just Pythagoras' theorem in disguise.

## Adding vectors

Vectors can be added in a similar way to scalars. Graphically, adding a vector  $\mathbf{b}$  to a vector  $\mathbf{a}$  involves drawing  $\mathbf{a}$  then drawing  $\mathbf{b}$  from the end of  $\mathbf{a}$ .

In the diagram to the right, adding  $\mathbf{b}$  to  $\mathbf{a}$  gets you from the origin,  $O$ , to point  $P$ .

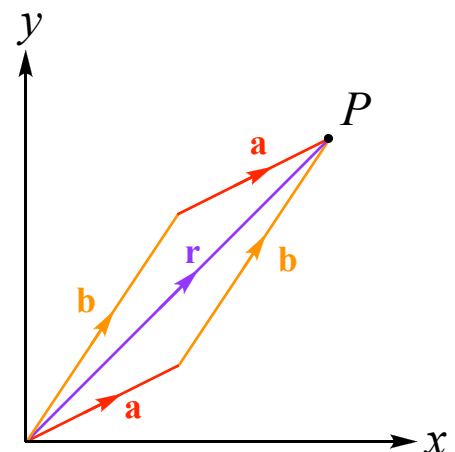
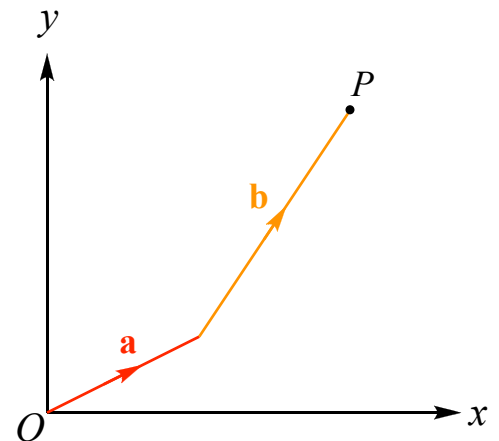
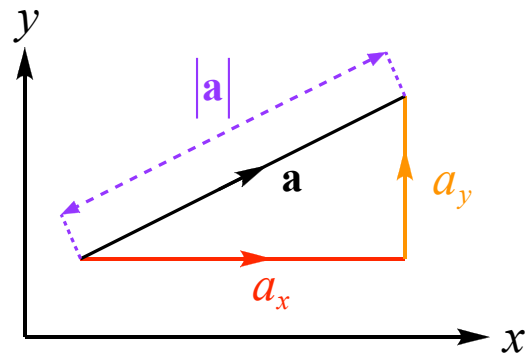
The result of adding to vectors is another vector. This is called the **resultant** vector.

In symbols,  $\mathbf{a} + \mathbf{b} = \vec{OP}$ .

Vector addition is commutative. This means the order in which you add two vectors doesn't matter; the resultant vector is the same.

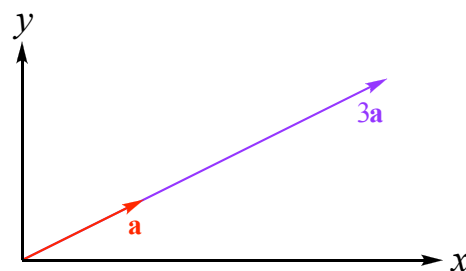
In the diagram to the right, you can see that  $\mathbf{a} + \mathbf{b}$  and  $\mathbf{b} + \mathbf{a}$  give exactly the same resultant vector.

I've denoted the resultant vector in this example by the symbol  $\mathbf{r}$ .



## Multiplying a vector by a scalar

You can multiply a vector by a scalar. For example, if you multiply a vector by 3, it becomes 3 times as long but points in the same direction.



We can describe **a** by the column vector  $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ .

Multiplying **a** by 3 gives **3a**, which is  $3 \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \times 2 \\ 3 \times 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$ , so  $3\mathbf{a} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$ .

In general, multiplying a vector **v** by a scalar **s** works as follows:

$$s\mathbf{v} = s \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} sv_x \\ sv_y \end{bmatrix}$$

	<p>Multiplying a vector by <math>-1</math> makes the vector point in the opposite direction.</p>
	<p>Multiplying a vector by <math>-2</math> makes the vector point in the opposite direction and makes it twice as long.</p>
	<p>Multiplying a vector by a scalar less than 1 makes the vector shorter. For example, multiplying a vector by <math>\frac{1}{2}</math> makes it half as long.</p>