## Trigonometry

## What's trigonometry?

Trigonometry is a method for finding sides or angles in a right-angled triangle. It consists of three formulae that are used in three different situations.

| What you know | What you want to find | What you use |
| :--- | :--- | :--- |
| two sides | the third side | Pythagoras' theorem |
| two sides | an angle | trigonometry |
| one side and an angle | another side | trigonometry |

In a right-angled triangle, one of the angles is always $90^{\circ}$. You need to know another angle to use trigonometry.

In fact, if you know one of the non- $90^{\circ}$ angles, you can calculate the other: you just subtract the angle you know from $90^{\circ}$.


## Labelling your triangle

In order to know which of the three formulae to use in a particular situation, you first need to label your triangle carefully. There are names and symbols for each of the three sides.

The longest side (which also happens to be the side opposite the right angle) is called the hypotenuse. This name is also used in Pythagoras' theorem.


| side | symbol | definition |
| :--- | :--- | :--- |
| hypotenuse | $h$ | longest side <br> side opposite the right angle |
| opposite | $O$ | side opposite angle $\theta$ |
| adjacent | $a$ | side next to angle $\theta$ that is not the hypotenuse |

## What's $\theta$ ?

The symbol $\theta$ is often used to represent an angle. This looks like a tablet or an egg sliced in half, but it is actually the Greek letter theta. Our alphabet (the Roman alphabet) has no letter equivalent to theta, but it is usually transliterated as $t h$. In fact, the Pythagoras' own name is spelt with a letter theta, since he was Greek: ПuӨaүópas.

## What are the formulae?

When doing trigonometry, you always know two things and want to find a third, unknown thing. There are three formulae to choose from:

| $\sin \theta=\frac{o}{h}$ | $\cos \theta=\frac{a}{h}$ | $\tan \theta=\frac{o}{a}$ |
| :--- | :--- | :--- |

A simple way to remember these rules is the mnemonic SOH CAH TOA, where S stand for $\sin \theta$, O stands for opposite and so on.

It is important to understand that $\sin \theta$ does not mean $\sin \times \theta$ as you would expect. It is actually a kind of number machine. You put in an angle between $0^{\circ}$ and $90^{\circ}$ and you get out a number between 0 and 1 .

$$
32^{\circ} \rightarrow \text { sine } \rightarrow 0.5299
$$

This is normally written as

$$
\sin 32^{\circ}=0.5299
$$

## How do I use trigonometry?

You are given a triangle whose longest side is 10 cm . Another side is 7 cm long, and you need to find the angle between the two known sides.

First of all, label your triangle:


As you can see from the labelled triangle, $h=10 \mathrm{~cm}$ and $a=10 \mathrm{~cm}$.
You need the formula that contains $h$ and $a$ :

$$
\cos \theta=\frac{a}{h}
$$

If you substitute in the values for $a$ and $h$ given in the diagram, you get $\cos \theta=\frac{7 \mathrm{~cm}}{10 \mathrm{~cm}} \quad$ therefore $\cos \theta=0.7$

We need to find $\theta$, not $\cos \theta$. The next step requires a calculator. You use the $\cos ^{-1}$ button to convert $\cos \theta$ to $\theta$ :

$$
\cos \theta=0.7 \quad \text { therefore } \quad \theta=\cos ^{-1}(0.7)
$$

In order to find $\theta$, you have to use a calculator to find $\cos ^{-1}(0.7)$.

Normally, you key in something like this:
 When you press $=$, the result you get is the angle $\theta$ :

$$
\cos ^{-1}(0.7)=45.572996^{\circ}
$$

You almost always round this value to the nearest decimal place or two.

$$
\cos ^{-1}(0.7)=45.6^{\circ} \text { to } 1 \text { decimal place }
$$

## Rearranging the formulae for different situations

There are nine possible situations in which you can use trigonometry. Each one requires a formula to be rearranged in a slightly different way. The trick is to get the unknown quantity alone on one side of the equals sign, with everything else on the other side.

All nine formulae shown are just rearrangements of the three main formulae we tend to refer to as SOHCAHTOA.
$\theta=\sin ^{-1}\left(\frac{o}{h}\right) \quad \theta=\cos ^{-1}\left(\frac{a}{h}\right) \quad \theta=\tan _{\substack{a \\ \text { unknown }}}^{\substack{\theta \\ \text { known }}}$

| $o=h \sin \theta$ | $a=h \cos \theta$ | $a=o \tan \theta$ |
| :---: | :---: | :---: |


|  |  |  |
| :---: | :---: | :---: |
| $h=\frac{o}{\sin \theta}$ | $h=\frac{a}{\cos \theta}$ | $o=\frac{a}{\tan \theta}$ |

