Rearranging

What's rearranging?

Rearranging is algebraic surgery! You take an equation and fiddle with it until it looks better.

Rearrangement changes an equation from one form to another. You often need to change the form of an equation in order to solve it or plot its graph.

$$\frac{5t+3}{4} = 1 \xrightarrow{\text{rearrangement}} t = \frac{1}{5}$$

How do I rearrange?

Rearranging always involves **doing the same thing to both sides** of an equation. That means whatever you do to the algebra on the left hand side of the equals sign, you have to do to the algebra on the right hand side, too.

Imagine what would happen if you didn't do the same thing to both sides. You'd get equations that made no sense or were impossible.

Start with this equation:

$$x + x = 2x$$

Add 1, but only to the right hand side. You get:

$$x + x = 2x + 1$$

This is impossible. See what happens when x = 3:

$$x + x = 2x + 1$$

$$3 + 3 = 2 \times 3 + 1$$

$$6 = 6 + 1$$

$$6 = 7$$

We can't have that! The only way to ensure that you are not making equations say things they are not meant to say is to always do the same thing to both sides.

Show me some rearranging in action

Let's rearrange $3x+5=11$ to find the value of *x*.

What can I do to both sides that gets me closer to the kind of answer I want? Ask yourself this question when rearranging. You need to know what you want your answer to look like. Here, we want the value of *x*, so we need an equation that looks like this:

$$x = \text{some number}$$

We need *x* on its own on one side of the equals sign, and a plain number on the other side.

$$3x+5=11$$

$$\boxed{-5}\ \downarrow \qquad \downarrow\ \boxed{-5}$$

$$3x=6$$

$$\boxed{\div 3}\ \downarrow \qquad \downarrow\ \boxed{\div 3}$$

$$x=2$$

Step one: remove constants

To get *x* on its own, we first need to get any constants (numbers added to or subtracted from *x*) off the left hand side.

The constant in $3x+5=11$ is +5, so to peel it away from the left hand side, we have to do the exact opposite: −5. This must be done to both sides. The reason we subtract 5 is to convert $3x+5$ to $3x$. This means 11 must be converted to 6.

Step two: remove coefficients

We now need to convert $3x$ to x. $3x$ means *x* multiplied by 3. The opposite of multiplication is division, so to get x we have to divide both sides by 3.

On the left hand side, we get what we need: *x* on its own. On the right hand side, we are left with 2. The equation is now in the form we need: $x = \text{some number}$.

Each step in rearranging involves noticing how the kind of equation you have is different from the kind you need, and then doing something mathematical to both sides to get closer to the kind of equation you need.

Most rearranging is removing numbers

When rearranging, you usually need to get a symbol on its own on one side of the equals sign. To do this, you need to counteract terms such as +3, −67, ÷4 and × ⅔.

The table below shows how to convert one side of an equation to the form you want. Always remember to do the same thing to both sides.

The left two columns give the general form of a conversion (a, b and c are just any old numbers). The right two columns give concrete examples.

Conversion required	**What you do to both sides**	**Example**
$ax+b \rightarrow ax$	$-b$	$10+3x=4 \xrightarrow{\boxed{-10}} 3x=-6$
$ax-b \rightarrow ax$	$+b$	$0.75x-5.4=1.3 \xrightarrow{\boxed{+5.4}} 0.75x=6.7$
$ax \rightarrow x$	$\div a$	$6y=18 \xrightarrow{\boxed{\div 6}} y=3$
$\frac{x}{c} \rightarrow x$	$\times c$	$\frac{t}{17}=3 \xrightarrow{\boxed{\times 17}} t=51$ $\frac{5x-2}{7}=2 \xrightarrow{\boxed{\times 7}} 5x-2=14$
$x^2 \rightarrow x$	$\sqrt{\ }$	$p^2=400 \xrightarrow{\boxed{\sqrt{\ }}} p=20$ $9x^2=64 \xrightarrow{\boxed{\sqrt{\ }}} 3x=8$
$x^3 \rightarrow x$	$\sqrt[3]{\ }$	$z^3=8 \xrightarrow{\boxed{\sqrt[3]{\ }}} z=2$

Formulae often need rearranging

A formula is a kind of equation. You usually need to rearrange it before you put in numbers. The reason you rearrange is to put all the symbols you have numbers for on one side, while keeping the symbol you don't know the value of on the other side. Then you can substitute in all the values you know, and the value of the symbol you don't know will appear.

Example from GCSE Maths

The formula for the sum of interior angles, *s*, of a polygon with *n* sides is

$$s = 180^\circ(n-2)$$

You are told that $s = 900^\circ$ and you are asked to find *n*, the number of sides the polygon has. The rearranging required is shown below:

$$s = 180^\circ(n-2) \xrightarrow{\boxed{\div 180^\circ}} \frac{s}{180^\circ} = n-2 \xrightarrow{\boxed{+2}} \frac{s}{180^\circ} + 2 = n$$

Substituting $s = 900^\circ$ into $\dfrac{s}{180^\circ} + 2 = n$ gives

$$\frac{900}{180^\circ} + 2 = n \quad \therefore \quad 5 + 2 = n \quad \therefore \quad n = 7$$

Example from GCSE Science

An important equation in physics is the wave equation, $v = f\lambda$.

You are told that $v = 3 \times 10^8$ and that $\lambda = 5 \times 10^{-7}$.

You are asked to find f. You need an equation with f on one side and v and λ on the other.

$$v = f\lambda \xrightarrow{\boxed{\div \lambda}} \frac{v}{\lambda} = f$$

You substitute in the values you have for v and λ to obtain the value of f:

$$f = \frac{v}{\lambda} \quad \therefore \quad f = \frac{3 \times 10^8}{5 \times 10^{-7}} \quad \therefore \quad f = 6 \times 10^{14}$$