Quadratic formula

What is the quadratic formula?

The quadratic formula is a formula for solving quadratic equations.

Quadratic equations can be written in the form $ax^2 + bx + c = 0$.

The quadratic formula will give you two values of *x* that satisfy your quadratic equation – these values of *x* are the solutions to your quadratic.

The quadratic formula is provided in the formula sheet at the beginning of exam papers:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

How do I use the quadratic formula?

You'll need a quadratic equation to solve, and it must be written in the form $ax^2 + bx + c = 0$. If the quadratic isn't in this form, you have to rearrange it until it is in this form.

For example, you may want to solve have the following quadratic equation:

$$x^2 - 6x - 3 = 0$$

The first thing to do is identify the values of *a*, *b* and *c*. By inspection, you can see that

$$a = 1$$
$$b = -6$$
$$c = -3$$

Next, substitute these values into the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 1 \times (-3)}}{2 \times 1}$$

All you have to do now is complete the calculations in the formula.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 1 \times (-3)}}{2 \times 1}$$

$$x = \frac{6 \pm \sqrt{36 - (-12)}}{2}$$

$$x = \frac{6 \pm \sqrt{36 + 12}}{2}$$

$$x = \frac{6 \pm \sqrt{48}}{2}$$

$$x = \frac{6 \pm 4\sqrt{3}}{2}$$

$$x = \frac{6 + 4\sqrt{3}}{2} \quad \text{or} \quad x = \frac{6 - 4\sqrt{3}}{2}$$

$$x = 3 + 2\sqrt{3} \quad \text{or} \quad x = 3 - 2\sqrt{3}$$

$$x = 6.46 \quad \text{or} \quad x = -0.46$$

Problems you may encounter

The quadratic formula is very powerful, but it has its disadvantages.

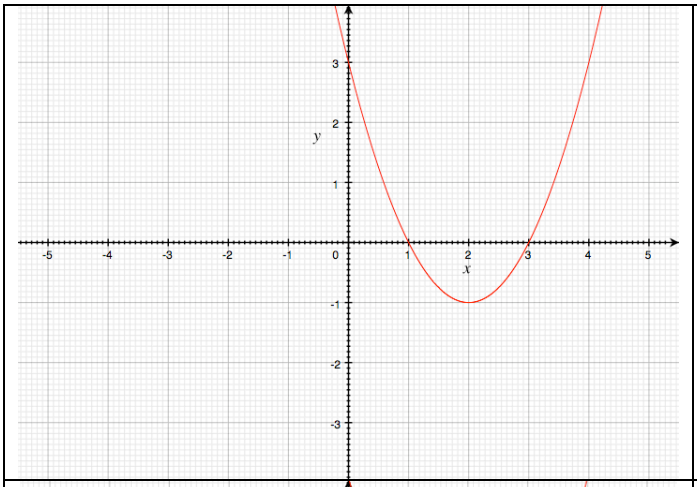
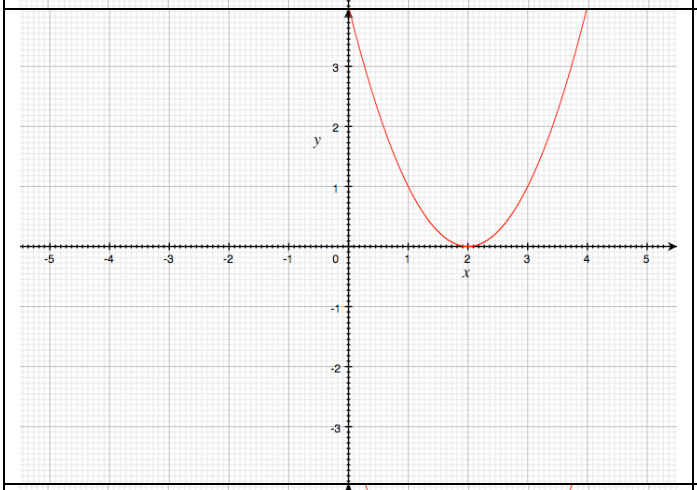
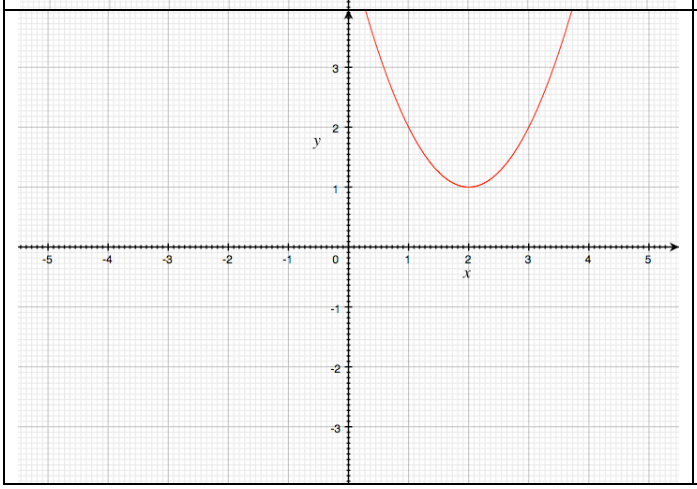
Using quadratic formula is hard work

The main problem is it takes quite a long time and quite a few steps to produce a solution. If you can solve a quadratic equation by factorising or completing square, it's probably quicker and easier to do so.

Some quadratic equations cannot be solved

Some quadratic equations have values of *a*, *b* and *c* such that $b^2 - 4ac$ is negative. It is impossible to find the square root of a negative number, so if you try to calculate $\sqrt{b^2 - 4ac}$ when $b^2 - 4ac < 0$, your calculator will return an error message of some kind, perhaps **Math Error** or similar.

The reason you can't find a solution to certain quadratics is because they don't all have solutions!

	$b^2 - 4ac > 0$ In this case, there are two solutions, so there are two points where the curve crosses the *x*-axis. **Example:** $x^2 - 4x + 3 = 0$
	$b^2 - 4ac = 0$ In this situation, there is only one solution to the quadratic equation, and only one *x*-intercept. The curve just touches the *x*-axis at this value of *x*. **Example:** $x^2 - 4x + 4 = 0$
	$b^2 - 4ac < 0$ Here, there are no solutions because you cannot find the square root of a negative number. **Example:** $x^2 - 4x + 5 = 0$