Proof in a nutshell

What's proof?

A proof uses **logic** to show a mathematical statement is **true in all cases**. All proofs need one or more **assumptions** to build on.

What's not proof?

Demonstrations are not proofs of statements with more than one possible case because a demonstration can only show a statement to be true for one case at a time.

Proofs you need to understand

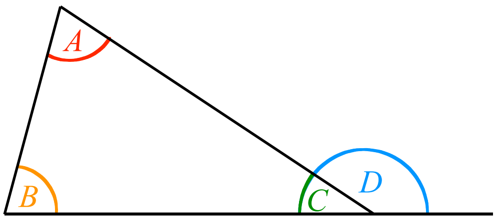
If you are taking intermediate maths, you need to **understand:**

- A proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices
- A proof that the sum of angles in a triangle is 180°

Exterior angle in triangle proof

Draw up a general triangle whose angles could take on any number of values. Represent the angles with symbols – I choose A, B, C and D.

We want to prove that the exterior angle of a triangle (D) is equal to the sum of the interior angles at the other two vertices (A and B). In equation form, this statement becomes D = A + B.



Assumptions:

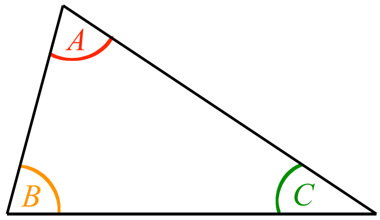
1. Angles on a straight line add up to 180°
2. Angles in a triangle add up to 180°

Based on assumption 1, **C + D = 180°**, therefore **C = 180° − D**.
Based on assumption 2, **A + B + C = 180°**.
Putting **180° − D** wherever **C** is gives **A + B + 180° − D = 180°**.
Subtracting 180° from each side gives **A + B − D = 0**.
Adding D to both sides gives **A + B = D**, which is what we wanted to prove.

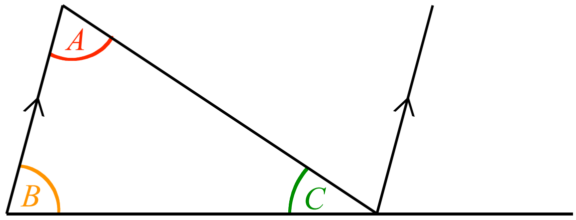
Mathematicians often write Q.E.D. after getting to the result required to prove a statement. Q.E.D. is an abbreviation for the Latin phrase *quod erat demonstrandum*, meaning, “that which was to be shown, has been shown”.

Proof that the sum of angles in a triangle is 180°

Start with a non-specific triangle that has sides of unspecified length and unspecified angles:

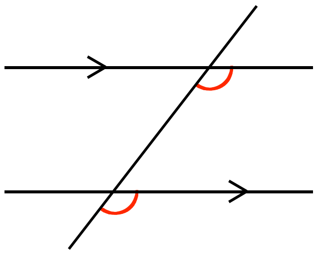


From one corner, draw a line parallel to one side of the triangle and extend the base of the triangle a little:

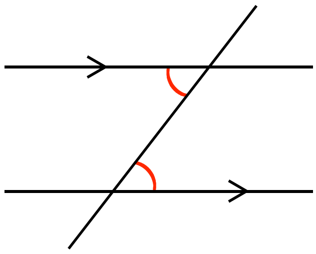


Assumptions

We now need two assumptions – two general rules about angles and parallel lines that we will assume are always true. If we wanted to prove the assumptions, we could do it later or let someone else prove them. However, you cannot get to a stage where there are no assumptions. All mathematical theorems (statements that have been proved) use assumptions. The assumptions we need in this case are:

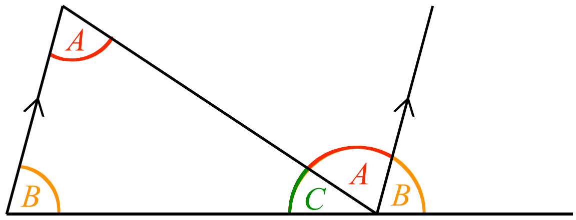


Corresponding angles (F-angles) are equal



Alternate angles (Z-angles) are equal

The assumptions we have made allow us to say that, if our assumptions are true, then the following diagram is labelled correctly.



We now need a third assumption – that angles on a straight line add up to 180°. On this basis, C + A + B = 180°. These are the same angles in the triangle, so angles in any triangle always add up to 180° (if our assumptions are correct).