## Circles

## What exactly is a circle?

This sounds like a stupid question, but circles do in fact have a mathematical definition:

## A circle is a set of points that are all a certain distance from a fixed point called the centre.

To construct a circle, you could use the following method:

| 1. Choose a centre <br> and mark it. | 2. Draw a straight <br> line from the centre <br> and mark a point at <br> a certain distance (I <br> used 2 cm$)$ <br> centre. from the | 3. Draw another <br> straight line from <br> the centre, in a <br> different direction. <br> Draw another point <br> that is 2 cm from <br> the centre. | 4. Keep adding <br> points in different <br> directions, making <br> sure they are all 2 <br> cm from the centre. | 5. When you've <br> added many points <br> in enough different <br> directions, they will <br> start to resemble a <br> circle. | 6. If you added an <br> infinite number of <br> points, you would <br> get a circle. To save <br> drawing points <br> forever, we use a <br> line. |
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|  |  |  |  |  |  |

From this method, you can see that every point on the circle's circumference is exactly 2 cm from the centre. Different circles have different distances from centre to circumference. Every point on the circumference must be exactly the same distance from the centre as every other point on the circumference.
This is what makes a circle a circle.

## What are those fancy words used to describe circles?

In maths, you often need a specific word to describe the part of a circle you mean. Otherwise you end up confusing everyone with phrases like that long curvy bit and that line thing. It's best to be specific!


## Calculating circumference

The diameter of a circle is exactly twice the length of its radius. We usually give the radius the symbol $r$ and the diameter the symbol $d$. The formula that connects the diameter and the radius of a circle is $d=2 r$.

You can also write this as $r=\frac{1}{2} d$ or $r=\frac{d}{2}$.
So, to convert radius to diameter, double it. To convert diameter to radius, halve it.


You can also calculate the circumference of a circle if you know its radius or its diameter.
Suppose you know the diameter of a circle. To find its circumference, all you do is multiply the diameter by $\pi$.
The formula that relates diameter to circumference is shown below. The symbol c represents circumference.
$c=\pi d$

You can rearrange this formula to get another formula, this time with $d$ as the subject:
$d=\frac{c}{\pi}$

You can also use the fact that the radius of a circle is half its diameter to write an equation connecting radius and circumference. All you do is replace $d$ with $2 r$.
$c=\pi d \quad$ so $\quad c=\pi(2 r) \quad$ which is usually written $\quad c=2 \pi r$

## What is $\pi$ ?

$\pi$ is a number whose approximate value is 3.141592654 . This is the value your calculator uses. A simpler value to remember is 3.14 . On the front of exam papers, they tell you to use 3.14 for $\pi$ if your calculator doesn't have a $\pi$ button.

You might be surprised to learn that $\pi$ has an infinite number of decimal places. Numbers like this are called irrational numbers.

## Calculating the area of a circle

You'll inevitably have to use this formula at least once in your exams: $A=\pi r^{2}$
Learn it off by heart, stick a poster of it on the ceiling above your bed, carve it into your wooden leg... just make sure you remember it!

The syllabus says you should be able to rearrange formulae like this one. So here goes. Suppose you know the area of a circle, but not its radius. To find the radius, you have to rearrange the area-of-a-circle formula to make $r$ the subject - to make it look like $r=$ something.

Start by writing out the formula: $\quad A=\pi r^{2} \quad$. Divide both sides by $\pi$ : $\frac{A}{\pi}=r^{2}$. Square root both sides: $\quad \sqrt{\frac{A}{\pi}}=r$
All you have to do now is fill in the values. If the circle has an area of $100 \mathrm{~mm}^{2}$ then its radius is calculated as follows:

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r=\sqrt{\frac{A}{\pi}}=\sqrt{\frac{100}{3.14}}=31.8 \mathrm{~mm}
$$

